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MART Program, Phase I, Radiation Theory

Technical Note

INFINITE ARRAYS WITH NONRIGID INTERSTICES

V. MANGULIS

CAI/SED-7105 Report No. I-TN-1
Contract No. N00024-69-C-1342

Computer Applications Incorporated

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ABSTRACT

The radiation from an infinite phased planar array of rectangular pistons (all with the same velocity magnitude) in a nonrigid baffle is evaluated. The nonrigid interstices between the array elements are approximated by equivalent undriven pistons with some internal impedance. When the dimensions of the pistons are small compared to a wavelength, simple approximations are obtained for the radiation impedance of an array element and the loss in source level due to the effect of the interstices. The source level is multiplied by the factor $1/|1 + (\Delta A/Z_I \cos\theta)|^2$, where ΔA is the fraction of the array area occupied by interstices, θ is the angle between the normal to the array and the direction to which the array is phased, and Z_I is the interstitial impedance per unit area divided by ρc, ρ = density of water, c = velocity of sound in water.

ACKNOWLEDGMENT

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I. INTRODUCTION

When radiation from sonar arrays is considered, usually one prescribes the velocity distribution of the array elements, and one assumes that the baffle, if there is one, is rigid. With such assumptions the mathematical problem is frequently simpler than with some other possible assumptions. However, the interstitial areas between the array elements often are not rigid; in some arrays there is no baffle at all, in other arrays the baffle material may not be rigid enough. Nonrigid surfaces may have a profound effect upon the propagation of sound.[1-6] Consequently, a study of the effects of nonrigid interstices is of considerable importance in the design of sonar arrays.

We would like to obtain for the effects of interstices a mathematical solution which is simple and yet accurate. The mathematical modeling of finite arrays on finite baffles is complicated even if the baffles are rigid. However, the radiation of sound from an infinite phased array on a rigid plane baffle can be described by relatively simple approximate expressions under certain conditions, and the nearfield sound pressure has been shown to be roughly the same for infinite and large finite arrays for angles not too near to endfire.[7, 8]. Consequently, we have evaluated the effects of non-rigid interstices in infinite arrays; such effects are expected to approximate the effects in large finite arrays.

The nonrigidity of the interstitial baffle areas can be modeled in various ways; the simplest mathematical condition is to set the pressure on the interstice equal to the velocity of the interstitial baffle surface times an interstitial impedance per unit area. Such a condition implies that the nonrigid interstitial surface reacts only to the local pressure, and the movement of the surface is not affected by the acoustic field elsewhere. For example, if the interstitial volume between the sonar array elements is filled with water, such cavities may have a wide range of impedances, depending on the geometry of the cavities; in general the cavities may be coupled to each other, and therefore the movement of the interstitial surface may depend not only on the local pressure, but also on the pressure at the other cavities. We will assume that such coupling of cavities is weak enough so that at the interstices we can use the local boundary condition: pressure equals velocity times an impedance per unit area. Furthermore, if we break up the interstitial area into smaller areas with dimensions much less than a wavelength, then each smaller area will move with roughly the same velocity, and therefore can be regarded as a piston; i.e., the interstitial area between the active radiating pistons can be broken up into small equivalent passive undriven pistons with some internal impedance. The radiating pistons have a prescribed velocity distribution which is obtained by applying the appropriate voltages to the transducer elements. The equivalent interstitial pistons are vibrated by the acoustic field only; their velocity distribution depends on the mutual coupling between the interstitial and radiating pistons, and on the interstitial impedance. A similar approach has been used to model a nonrigid baffle surrounding a small array (not the interstitial baffle between the elements).[9,10]

Thus we will consider an infinite array of rectangular pistons with the interstices approximated by another set of rectangular pistons as shown in Figure 1. The radiating pistons are unshaded in Figure 1, and the interstitial pistons are shown shaded.

All interstitial pistons have the same internal impedance per unit area. To be specific, we are showing two interstitial pistons for each radiating piston in Figure 1; however, if desired, the interstitial area could be divided further into still smaller equivalent pistons.

In the next sections we will solve for the velocities of the interstitial pistons when the velocities of the radiating pistons are specified, and we will obtain the nearfield and farfield pressure and the radiation impedance of an element. Simple approximate expressions for the radiation impedance and the loss in source level will be obtained for the case when the dimensions of the radiating and interstitial pistons are much less than a wavelength. Numerical results will be presented for the approximate loss in source level.

II. EXACT EQUATIONS

Let the infinite array and the nonrigid baffle occupy the plane $z=0$ (see Fig. 1). The centers of the radiating pistons are located at $x=nd_x$, $y=md_y$, $n=0, \pm 1, \pm 2, \ldots, m=0, \pm 1, \pm 2, \ldots$. The velocity (in the z direction) of the radiating piston in the m^{th} row and the n^{th} column is forced to be

$$v_{mn}^R = v_R e^{i\omega t + i\gamma_{mn}} \tag{1}$$

where v_R is a complex constant, ω the radian frequency, t the time, and

$$\gamma_{mn} = -mkd_y \sin\theta \sin\phi - nkd_x \sin\theta \cos\phi, \tag{2}$$

where $k=\omega/c$, c the velocity of sound in water, and θ, ϕ are angles in spherical coordinates which determine the direction to which the farfield pressure maximum is steered.

Let the velocity of the horizontally oriented interstitial piston above the m^{th}, n^{th} radiating piston in Fig. 1 (with center at $x=nd_x$, $y=(m+\frac{1}{2})d_y$) be denoted by

$$v_{mn}^H = v_H e^{i\omega t + i\gamma_{mn}}, \tag{3}$$

and let the velocity of the vertically oriented interstitial piston to the right of the m^{th}, n^{th} radiating piston in Fig. 1 (with center at $x=(n+\frac{1}{2})d_x$, $y=md_y$) be denoted by

$$v_{mn}^V = v_V e^{i\omega t + i\gamma_{mn}} . \tag{4}$$

For simplicity we will refer to pistons with velocities v_{mn}^R, v_{mn}^H, or v_{mn}^V as

R-type, H-type, or V-type pistons.

While v_R is known, v_H and v_V are not, and they have to be obtained from two equations for a force balance as shown in Appendix A. The solution of the two simultaneous equations gives

$$\frac{v_H}{v_R} = - \frac{S_{RH}(S_{VV} + Z_I) - S_{RV}S_{VH}}{(S_{HH} + Z_I)(S_{VV} + Z_I) - S_{HV}S_{VH}} \tag{5}$$

$$\frac{v_V}{v_R} = - \frac{S_{RV}(S_{HH} + Z_I) - S_{RH}S_{HV}}{(S_{HH} + Z_I)(S_{VV} + Z_I) - S_{HV}S_{VH}} \tag{6}$$

where $\rho c Z_I$ is the internal impedance per unit area of both types (horizontal or vertical) of the equivalent interstitial pistons and Z_I is dimensionless; ρ is the density of water; and for J=R, H, or V, and K=R, H, or V, the dimensionless quantities S_{JK} are given by

$$S_{JK} = \frac{iA_J}{A} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{D(A_J)\, D(A_K) e^{iE(J,K)}}{T(n, m)} \tag{7}$$

where A_J is the area of the piston of type J(=R, H, or V), $A_C = d_x d_y = A_R + A_H + A_V$, $D(A_J)$ and $D(A_K)$ are related to the directivity of a rectangular piston of type J and K,

$$D(A_J) = \frac{\sin\left[\frac{1}{2}g(m)ka_y^J\right]}{\frac{1}{2}g(m)ka_y^J} \cdot \frac{\sin\left[\frac{1}{2}h(n)ka_x^J\right]}{\frac{1}{2}h(n)ka_x^J} \tag{8}$$

where a_x^J and a_y^J are the dimensions of the $J^{\underline{th}}$ piston in the x and y direction,

$$h(n) = \sin\theta \cos\phi + 2\pi n/kd_x \tag{9}$$

$$g(m) = \sin\theta \sin\phi + 2\pi m/kd_y \tag{10}$$

$$w(n,m) = \left\{ \left[h(n) \right]^2 + \left[g(m) \right]^2 \right\}^{\frac{1}{2}} \tag{11}$$

$$T(n,m) = \begin{cases} i\left\{ 1 - \left[w(n,m) \right]^2 \right\}^{\frac{1}{2}}, & \text{if } 0 \le w(n,m) \le 1; \\ \left\{ \left[w(n,m) \right]^2 - 1 \right\}^{\frac{1}{2}}, & \text{if } w(n,m) > 1. \end{cases} \tag{12}$$

$$E(R,R) = E(H,H) = E(V,V) = 0 \tag{13a}$$

$$-E(R,V) = E(V,R) = \tfrac{1}{2}h(n)kd_x \tag{13b}$$

$$-E(R,H) = E(H,R) = \tfrac{1}{2}g(m)kd_y \tag{13c}$$

$$-E(H,V) = E(V,H) = \tfrac{1}{2}h(n)kd_x - \tfrac{1}{2}g(m)kd_y \tag{13d}$$

For example, if we write out explicitly the equations for J = R, K = V, we obtain from Eq.(7):

$$S_{RV} = \frac{is_x s_y}{d_x d_y} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \frac{e^{-i\frac{1}{2}h(n)kd_x}}{T(n,m)} \cdot \frac{\sin\left[\frac{1}{2}g(m)ks_y\right]}{\frac{1}{2}g(m)ks_y} \cdot \frac{\sin\left[\frac{1}{2}h(n)ks_x\right]}{\frac{1}{2}h(n)ks_x} \cdot \frac{\sin\left[\frac{1}{2}g(m)kd_y\right]}{\frac{1}{2}g(m)kd_y} \cdot \frac{\sin\left[\frac{1}{2}h(n)kt_x\right]}{\frac{1}{2}h(n)kt_x} \right\} \tag{14}$$

As shown in Appendix A, the total pressure $\rho c v_R e^{i\omega t} P_T$ can be evaluated as

$$P_T = P_R + (v_H/v_R)P_H + (v_V/v_R)P_V \qquad (15)$$

where P_T, P_R, P_H, and P_V are dimensionless quantities, and for J = R, H, or V

$$P_J = \frac{iA_J}{A_C} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{D(A_J)e^{-ih(n)kx - ig(m)ky - T(n,m)kz + iE(J,R)}}{T(n,m)} \qquad (16)$$

where $D(A_J)$ is given by Eq.(8), and E(J,R) is given by Eq.(13).

The radiation impedance of a radiating piston is $\rho c s_x s_y Z_R$, where the dimensionless quantity Z_R is derived in Appendix A,

$$Z_R = S_{RR} + (v_H/v_R)S_{HR} + (v_V/v_R)S_{VR}, \qquad (17)$$

and S_{RR}, S_{HR}, and S_{VR} are given by Eq.(7).

III. APPROXIMATIONS

While the exact equation for the pressure field contains an infinite number of terms as in Eq.(16), it has been shown that for small pistons close together a single term (the m=0, n=0 term) is sufficient to approximate the exact equation. Thus, if the array spacings d_x and d_y are much smaller than a wavelength, then, as shown in Appendix B, we obtain the following approximate expression for the total radiated pressure

$$P_T \approx G.\frac{A_R}{A_C\cos\theta}.\exp(-ikx\sin\theta\cos\phi - iky\sin\theta\sin\phi$$

$$-ikz\cos\theta) = G.P_r, \tag{18}$$

and the radiation impedance of an element becomes

$$Z_R \approx \frac{A_R}{A_C\cos\theta}.\ G = Z_r\ .\ G, \tag{19}$$

where P_r and Z_r are the approximate pressure and impedance which one would have obtained for small pistons close together if the interstices had been rigid.[7] Consequently, G is the factor which gives the effect of the nonrigid interstices,

$$G = \frac{Z_I\cos\theta}{\Delta A + Z_I\cos\theta} = \frac{1}{1 + (\Delta A/Z_I\cos\theta)} = \frac{Z_I\cos\theta/\Delta A}{1 + (Z_I\cos\theta/\Delta A)} \tag{20}$$

where ΔA is the fraction of the array area occupied by the interstices,

$$\Delta A = (A_H + A_V)\ /\ A_C \tag{21}$$

Note that G approaches 1 as it should when either $\Delta A \to 0$ or $|Z_I| \to \infty$. For $\Delta A \to 1$ G is finite, but P_r and Z_r vanish because $A_R \to 0$. $G \to 0$ if $Z_I \to 0$, i.e., the array does not radiate if the interstices are perfect pressure release surfaces (at least in this approximation the pressure vanishes; there may be some of the smaller terms in the exact equation which do not vanish). Observe that G depends only on the combination $Z_I\cos\theta/\Delta A$, not on Z_I, θ, or ΔA separately. Z_I and ΔA are dimensionless, therefore $Z_I\cos\theta/\Delta A$ is also dimensionless.

When $\theta \to 90^o$, then $G \to 0$, but $P_r \to \infty$ and $Z_r \to \infty$ and the products GP_r and GZ_r remain finite. However, the behavior of the infinite array does not approximate the behavior of the finite array for angles θ too close to 90^o.

Note that

$$Z_r = A_R / A_C \cos\theta \tag{22}$$

is a pure resistance, and under the right conditions the presence of the nonrigid interstices will make Z_R complex. We are mainly interested in the change in the power output, proportional to the radiation resistance, and thus proportional to the real part of G,

$$\mathrm{Re}\,(G) = \frac{(R_I^2 + X_I^2)\cos^2\theta + \Delta A R_I \cos\theta}{(\Delta A + R_I\cos\theta)^2 + (X_I\cos\theta)^2} \tag{23}$$

where $R_I + iX_I = Z_I$.

On the other hand, the source level in the far field is proportional to $|P_T|^2$ and thus it is proportional to $|G|^2$,

$$|G|^2 = \frac{(R_I^2 + X_I^2)\cos^2\theta}{(\Delta A + R_I\cos\theta)^2 + (X_I\cos\theta)^2} \tag{24}$$

The difference

$$\mathrm{Re}\,(G) - |G|^2 = \frac{\Delta A\, R_I \cos\theta}{(\Delta A + R_I\cos\theta)^2 + (X_I\cos\theta)^2} \qquad (25)$$

is proportional to the power dissipated in the interstices; the difference vanishes when $R_I = 0$ because when the interstice impedance is purely reactive then the interstices cannot absorb power.

Figure 2 shows $|G|^2$ vs. $|Z_I\cos\theta/\Delta A|$ for three different conditions: 1) $R_I = 0$; 2) $R_I = X_I$; 3) $X_I = 0$. The source level is highest for $R_I = 0$, lowest for $X_I = 0$; i.e., for the same value of the interstice impedance magnitude, a pure resistance produces a greater loss of source level than a pure reactance because the interstice then not only modifies the radiated pressure but also absorbs power. Note also that $|G|^2$ is independent of the sign of X_I.

IV. CONCLUSIONS

We have derived the exact expressions for the pressure [Eq.(15)] and radiation impedance [Eq.(17)] for an infinite array of rectangular pistons with passive interstitial pistons interspersed among the radiating pistons. For simplicity we assumed that the nonrigid interstices can be approximated by two equivalent pistons for each radiating piston; however, we could have specified without much difficulty more interstitial pistons for each radiating piston. With two interstitial pistons we had to solve two simultaneous equations; with N interstitial pistons we would have to solve N simultaneous equations.

Due to a lack of funds we have not evaluated the exact equations numerically. However, for small pistons close together we have been able to obtain a very simple approximate expression [Eq.(20)] for the factor which modifies the pressure and radiation impedance because of the presence of the nonrigid interstices. The approximate expression shows that for angles near broadside the source level will be reduced by less than 4 db if the interstices occupy less than 50% of the array area and if the magnitude of the normalized dimensionless interstitial impedance $|Z_I|$ is greater than 1 (see Figure 2). Consequently, one should be able to design an array with interstices in such a way that the detrimental effects of the interstices are tolerable -- if the presence of pressure release materials and resonances in interstitial cavities is avoided, then one should be able to make $|Z_I|$ greater than 1. The reduction of the area occupied by interstices would also help.

APPENDIX A.

DERIVATION OF THE EXACT EQUATIONS

Since the piston velocities and the pressure field at the $m^{\underline{th}}$ row and the $n^{\underline{th}}$ column will be the same as at m = 0 and n = 0, except for the phase γ_{mn}, we can confine our attention to the pistons at m = 0, n = 0.

First consider the pressure field due to the radiating pistons alone, i.e., assume the interstices are rigid. Then the pressure field can be obtained from Eq.(14) in Reference 7; in Reference 7 the pistons are circular, therefore the integration in Eq.(14) of Reference 7 is over the circular area of a piston; we must replace that by integration over the rectangular area of our piston, which then yields the dimensionless quantity P_R as given by Eq.(16) in Section II here.

Next let us calculate the pressure $\rho c v_H e^{i\omega t} P_H$ when the H-type pistons vibrate with velocities v_{mn}^H and the rest of the xy-plane is rigid. P_H is given by the same equation as P_R, if we replace the dimensions of the radiating pistons by the dimensions of the H-type pistons, and if we compensate for a shift of origin from the R-type to H-type pistons; the latter accounts for the phase E(H,R) in Eq.(16). P_V is obtained the same way.

Thus, if we know the velocities v_R, v_H, v_V, then the total pressure is obtained from the superposition of the three partial pressures due to each type of pistons:

$$\rho c v_R e^{i\omega t} P_T = \rho c e^{i\omega t} (v_R P_R + v_H P_H + v_V P_V). \tag{A-1}$$

The superposition yields the correct expression because, for example, P_H gives the correct velocities for the H-type pistons, and it adds nothing to the velocities of the R-type or V-type pistons, since P_H is a solution of the model where only H-type pistons vibrate, and R-type and V-type pistons are immobile. Similar considerations apply to P_R and P_V.

However, this far we still have not found the relationship between v_R and v_H, v_V. To obtain that relationship, we must consider the forces on the H-type and V-type pistons, i.e., the boundary condition on the interstices.

To find the total force on an H-type piston, we must integrate the total pressure, given by Eq.(A-1), over the area A_H. Define

$$S_{JK} = \frac{1}{A_K} \iint_{A_K} dA \, P_J, \tag{A-2}$$

i.e., the dimensionless quantity S_{JK} is the integral of the partial pressure due to all J-type pistons over the area of one K-type piston, divided by the area A_K; the integration yields Eq.(7) in Section II. S_{JK} is proportional to the force on one K-type piston due to the combined action of all J-type pistons. Then the total force F_H on the area A_H is

$$F_H = \rho c A_H e^{i\omega t}(v_R S_{RH} + v_H S_{HH} + v_V S_{VH}) \tag{A-3}$$

However, if the internal impedance of the H-type piston is $\rho c A_H Z_I$, then the boundary condition on the interstice (force = velocity times impedance) becomes

$$F_H = - v_H e^{i\omega t} \cdot \rho c A_H Z_I \tag{A-4}$$

where the minus sign arises from the fact that v_H is defined as the velocity into the water, while the boundary condition contains the opposite velocity (into the equivalent piston). If we equate Eqs.(A-3) and (A-4), we obtain

$$v_R S_{RH} + v_H S_{HH} + v_V S_{VH} = - v_H Z_I . \tag{A-5}$$

Similarly, from the total force F_V on one V-type piston with internal impedance $\rho c A_V Z_I$ we find

$$v_R S_{RV} + v_H S_{HV} + v_V S_{VV} = - v_V Z_I \tag{A-6}$$

If we solve simultaneously Eqs.(A-5) and (A-6), then we obtain

Eqs.(5) and (6) in Section II. With v_H and v_V specified, the total pressure as given by Eq.(A-1) is now completely determined.

Moreover, from the total force F_R on one radiating piston we obtain the radiation impedance $\rho c A_R Z_R$, because by definition

$$F_R = \rho c A_R Z_R . v_R e^{i\omega t}, \tag{A-7}$$

and, furthermore, from the integral of the total pressure on the radiating piston,

$$F_R = \rho c A_R e^{i\omega t}(v_R S_{RR} + v_H S_{HR} + v_V S_{VR}), \tag{A-8}$$

consequently, if we equate Eqs.(A-7) and (A-8), we obtain Eq.(17) for the normalized dimensionless radiation impedance Z_R.

APPENDIX B

DERIVATION OF THE APPROXIMATE EXPRESSIONS

If $kd_x \to 0$ and $kd_y \to 0$ (and then $ks_x \to 0$, etc., because $s_x \leq d_x$, etc.), then from Eqs.(9) and (10) $h(n) \to \infty$ and $g(m) \to \infty$, except for $n = 0$ and $m = 0$. Furthermore, $T(n,m) \to \infty$, except that

$$T(0,0) = i\cos\theta \tag{B-1}$$

Consequently, for small pistons close together the dominant terms in Eqs.(7) or (16) are the terms with $m = 0$, $n = 0$. For $m = 0$, $n = 0$, and small pistons close together the directivity $D(A_J) \approx 1$ in Eq.(8), and the $E(J,K) \to 0$. Thus

$$S_{JK} \to A_J/A_C \cos\theta. \tag{B-2}$$

$$\frac{v_H}{v_R} \rightarrow \frac{v_V}{v_R} \rightarrow \frac{-A_R}{A_H + A_V + A_C Z_I \cos\theta} \tag{B-3}$$

$$P_J \rightarrow (A_J/A_C \cos\theta) \exp(-ikx\sin\theta\cos\phi - iky\sin\theta\sin\phi - ikz\cos\theta) \tag{B-4}$$

If we substitute Eqs.(B-2), (B-3), and (B-4) into Eqs.(15) and (17), then we obtain Eqs.(18) and (19).

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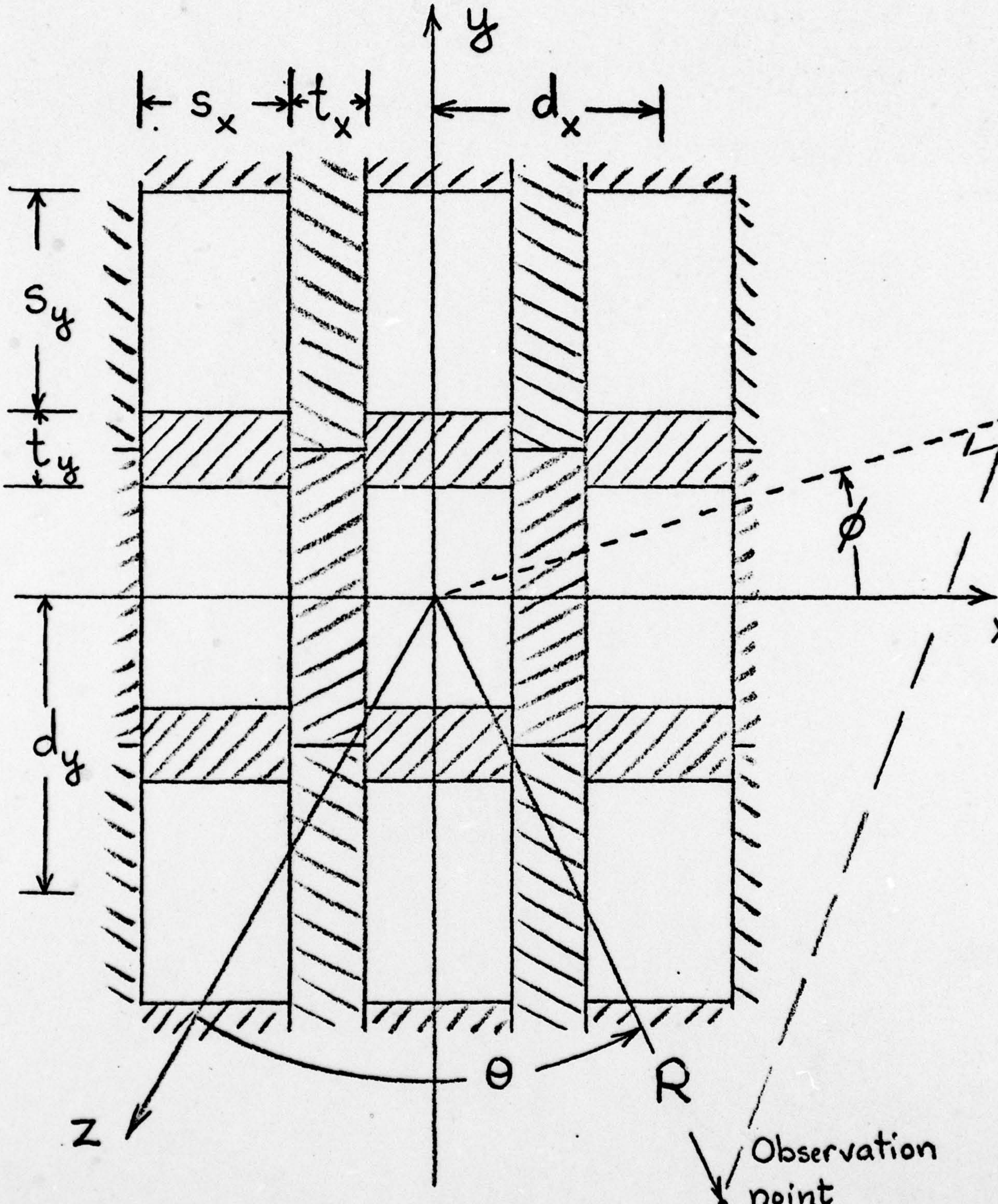

Fig. 1. A part of the infinite array.

Figure 2. Relative source level vs. $|Z_I \cos\theta/\Delta A|$